THERMOELASTIC SLOPING THIN-WALLED SHELL STRUCTURES DEFORMED WITHOUT BENDING

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We have derived a nonlinear integrodifferential equation to find the unbent shape of the middle surface of a sloping shell of constant thickness, subjected to a given temperature and force field.

The problem of finding thin-walled shell structures with unbent shapes can be simulated by three groups of equations representing the Hooke law, the equations of equilibrium, and the conditions of frame compatibility for the zero-moment state. In the general case these equations are complex in form and the most complete results are therefore obtained for axisymmetrically deformed shells of revolution, Monge shells with flat parallels and meridians and sloping transition shells. In the following we offer an analysis of this class of problems for sloping shells, based on the utilization of appropriate simplifications.

Let the equation for the middle surface of the shell have the form

$$z = f(x, y), \ (x, y) \in D, \tag{1}$$

where

$$\left| \frac{\partial f}{\partial x}(x, y) \right| \ll 1, \quad \left| \frac{\partial f}{\partial y}(x, y) \right| \ll 1. \tag{2}$$

Conditions (2) indicate that the coefficients of the first quadratic form of the surface (1) are close to unity and on differentiation behave as constants [1]. The above-cited system of equations can therefore be represented in the following form:

$$\frac{\partial T_1}{\partial x} + \frac{\partial S}{\partial y} = -X, \tag{3}$$

$$\frac{\partial T_2}{\partial y} + \frac{\partial s}{\partial x} = -Y,$$

$$k_1 T_1 + k_2 T_2 = Z;$$

$$\frac{\partial^2 \varepsilon_2}{\partial x^2} + \frac{\partial^2 \varepsilon_1}{\partial y^2} = \frac{\partial^2 \omega}{\partial x \partial y}; \tag{4}$$

$$\varepsilon_1 = \frac{1}{Eh}[T_1 - \mu T_2 + c\theta];$$

$$\varepsilon_2 = \frac{1}{Eh}[T_2 - \mu T_1 + c\theta]; \tag{5}$$

$$\omega = \frac{2(1+\mu)}{Eh} S,$$

$$k_1 = -\frac{\partial^2 f}{\partial x^2}(x, y), \quad k_2 = -\frac{\partial^2 f}{\partial y^2}(x, y). \tag{6}$$

Through the obvious transformations from (3)-(5) at constants E, μ, h we obtain:

$$\Delta(T_1 + T_2) = -(1+\mu)\left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y}\right) - c\Delta\theta \equiv \varphi(x, y), \tag{7}$$

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$$k_1T_1 + k_2T_2 = Z, \quad (8)$$

$$\frac{\partial^2 T_1}{\partial x^2} - \frac{\partial^2 T_2}{\partial y^2} = \frac{\partial Y}{\partial y} - \frac{\partial X}{\partial x}. \quad (9)$$

Therefore

$$T_1 = \frac{1}{k_1 - k_2}\{Z + k_2F\}, \; T_2 = \frac{1}{k_2 - k_1}\{Z + k_1F\}, \quad (10)$$

where

$$F(x, y) = \iint_D \varphi(x_0, y_0) G(x, y; x_0, y_0)\, dx_0 dy_0 + \int_\Gamma \psi(x_0, y_0) \frac{\partial G(x, y; x_0, y_0)}{\partial n} dl. \quad (11)$$

Here $G(x, y; x_0, y_0)$ is the Green's function of the Dirichlet problem for region D; ψ is the value of $T_1 + T_2$ at the boundary D. In specific problems the determination of $T_1 + T_2$ on $\Gamma = \partial D$ calls for the application of Eq. (8) to the standard specified conditions of subjecting the edges of the shell to force loading

Incorporating (10) into (9), in order to find the unknown unbent shape of the middle surface of the shell we obtain the following integrodifferential equation:

$$\frac{\partial^2}{\partial x^2}\left[\frac{1}{k_1 - k_2}(Z + k_2F)\right] + \frac{\partial^2}{\partial y^2}\left[\frac{1}{k_1 - k_2}(Z + k_1F)\right] = \frac{\partial Y}{\partial y} - \frac{\partial X}{\partial x}. \quad (12)$$

On the basis of the usual assumptions from the theory of sloping shells [2] and with consideration of (6) from (12) we have

$$f''_{xx}(F''_{yy} - Y'_y + X'_x) + f''_{yy}(F''_{xx} + Y'_y - X'_x) = Z''_{xx} + Z''_{yy}. \quad (13)$$

The primes here indicate derivatives with respect to the variables indicated at the bottom of the corresponding functions.

Thus, determination of the sought shape of the middle surface (1) reduces to the solution of Eq. (12) or the equivalent Eq. (13). The effective realization of these equations in actual practice is associated with the possibility of constructing the functions $G(x, y; x_0, y_0)$ explicitly, and for a broad class of regions D these can be constructed by means of the following formula:

$$G(x, y; x_0, y_0) = -\frac{1}{2\pi}\operatorname{Re}\left[\lg \frac{w(z) - w(z_0)}{1 - w(z)w(z_0)}\right],$$

where $z = x + iy$, $z_0 = x_0 + iy_0$; $w(z)$ is the function which renders a conformal mapping of the region D onto a circle of unit radius. In particular, for a rectangle $0 < x < a$, $0 < y < b$ it has the form

$$G(x, y; x_0, y_0) = -\frac{1}{2\pi}\lg\left|\frac{\sigma(z - x_0 - iy_0)\sigma(z + x_0 + iy_0)}{\sigma(z - x_0 + iy_0)\sigma(z + x_0 - iy_0)}\right|,$$

where $\sigma(z)$ is the Weierstrass function:

$$\sigma(z) = \sigma(z, 2a, 2bi) = z\prod_{m,n}\left(1 - \frac{z}{2am + 2bni}\right)\exp\left(\frac{z}{2am + 2bni} + \frac{z^2}{8(am + bni)^2}\right), \; m^2 + n^2 \neq 0.$$

The Green's functions for a circle and the sector of a circle are of simpler form.

In conclusion, let us note that the final procedure for finding the function $z = f(x, y)$ can be completed by resorting to finite-difference approximation of Eqs. (12) or (13).

NOTATION

X, Y, Z, components of the external loading surface; $T_1 = T_x$, $T_2 = T_y$, S, generalized forces acting in the normal cross sections of the shell; D, projection of the shell onto the plane x0y; $\Gamma = \partial D$, boundary of the region D; $\varepsilon_1 = \varepsilon_x$; $\varepsilon_2 = \varepsilon_y$, ω, components of shell deformation; k_1, k_2, principal curvatures of the middle shell surface; $z = f(x, y)$, equation of the middle surface of the shell; E, μ, Young's modulus and Poisson coefficient for the shell material; h, shell thickness; $\theta = \theta(x, y)$, change in the temperature field of the shell.

LITERATURE CITED

1. S. A. Ambartsumyan, *The General Theory of Anisotropic Shells* [in Russian], Moscow (1974).
2. V. L. Biderman, *The Mechanics of Thin-Walled Structures. Statics* [in Russian], Moscow (1977).